Conditioning in (non probabilistic) graphical models

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Graphical models

Graphical models are important tools for :

- Reasoning and decision under uncertainty

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- Learning (passive data vs manipulation experiments)

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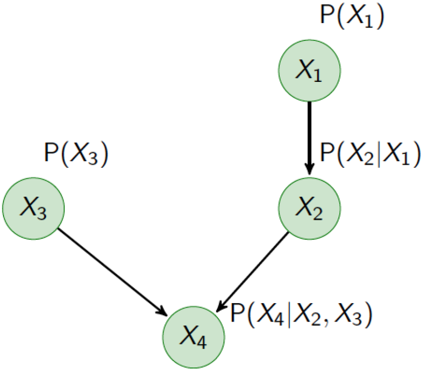
Local distributions:
conditional probability
$P(X_i|Pa(X_i))$
for each node (X_i)
given its parents $Pa(X_i)$



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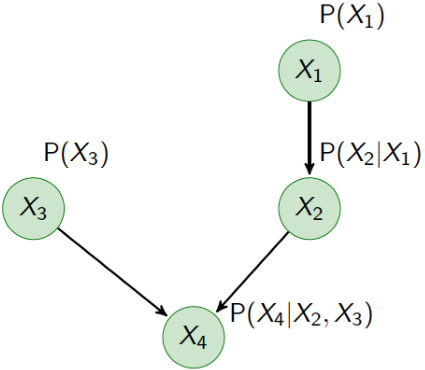
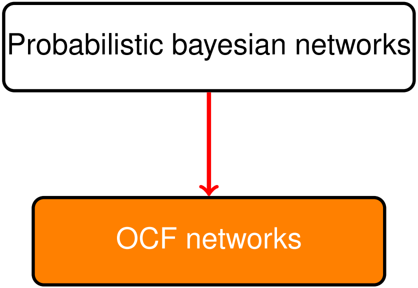


- Local probability distributions induce a unique joint distribution :

$$P(X_1, X_2, X_3, X_4) = P(X_1).P(X_2|X_1).P(X_3).P(X_4|X_2, X_3)$$

Non-probabilistic graphical models



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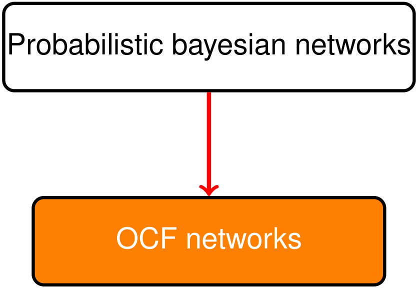

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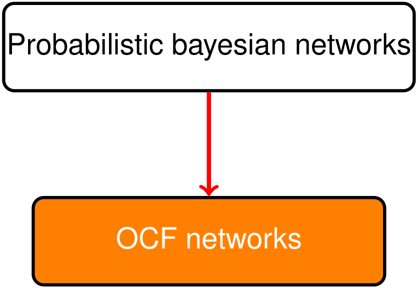

- Ordinal Conditional Functions (OCF or κ functions)

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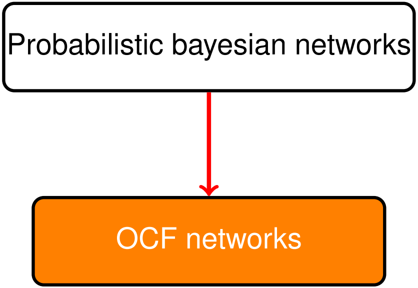

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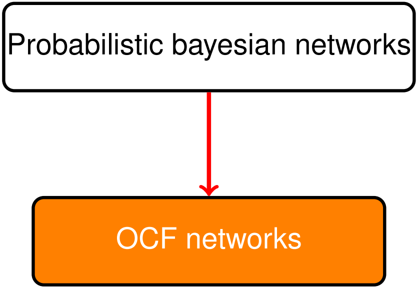

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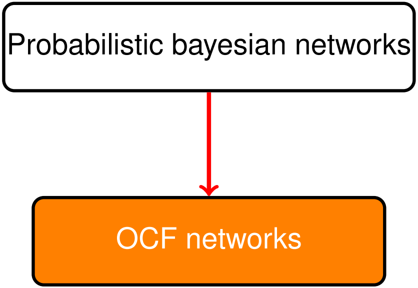

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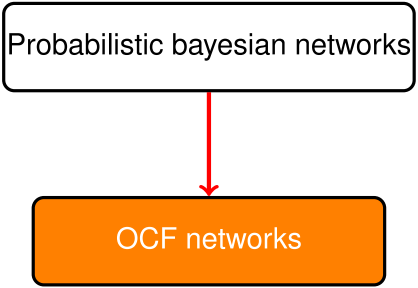

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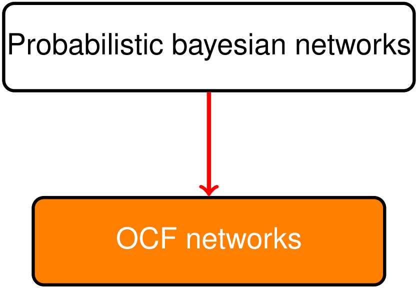

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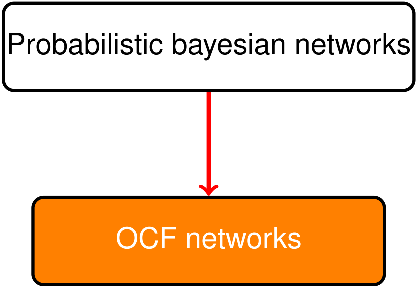

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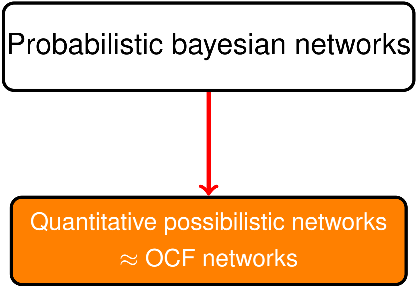

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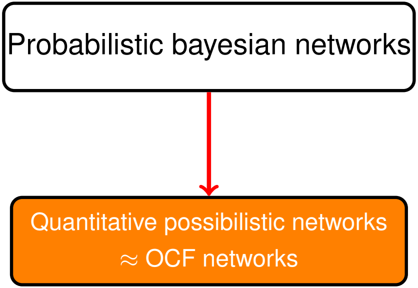

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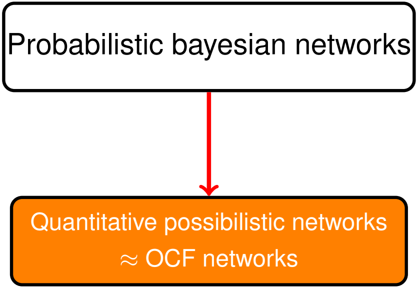

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Non-probabilistic graphical models

Probabilistic bayesian networks

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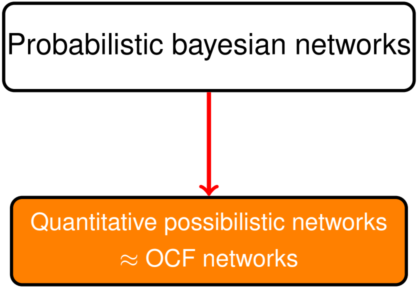

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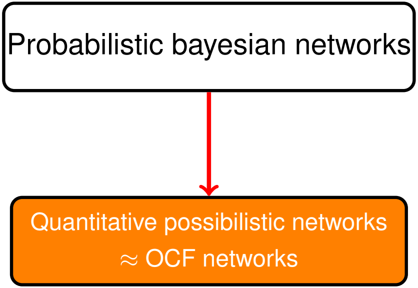

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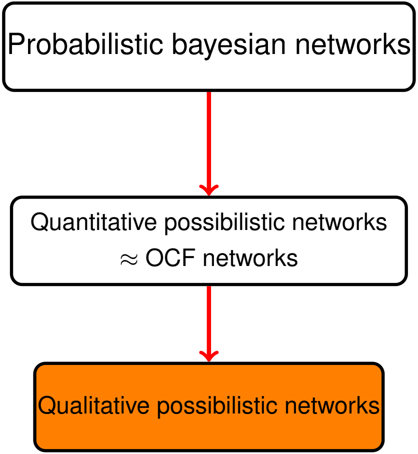

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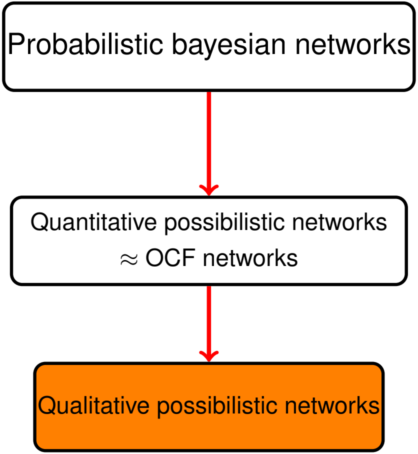

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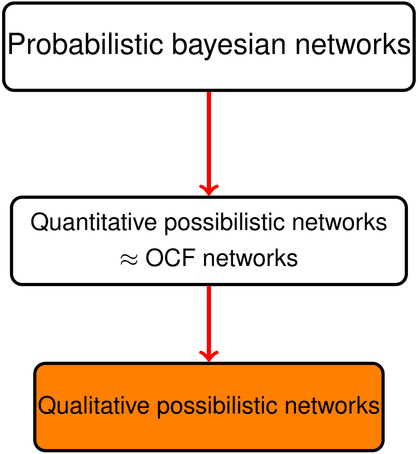

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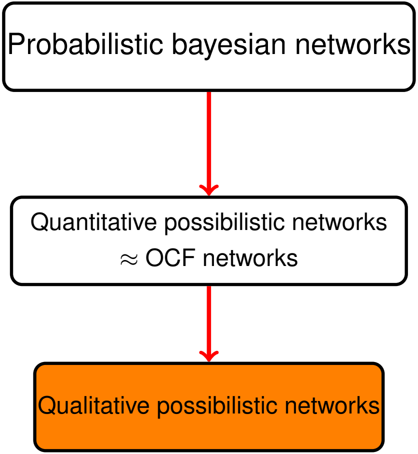

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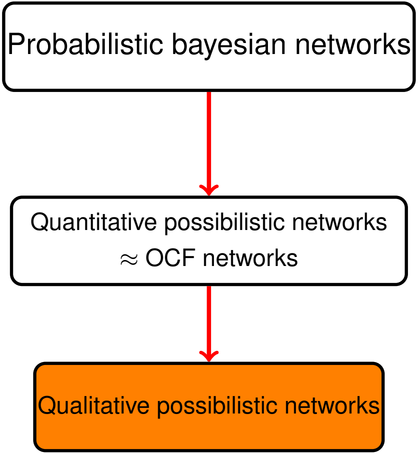

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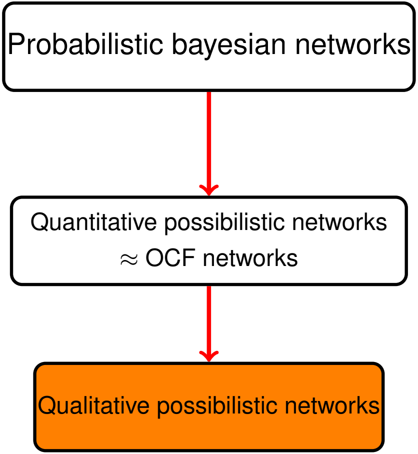

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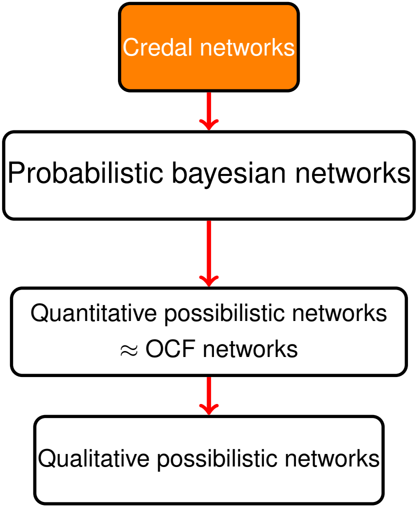

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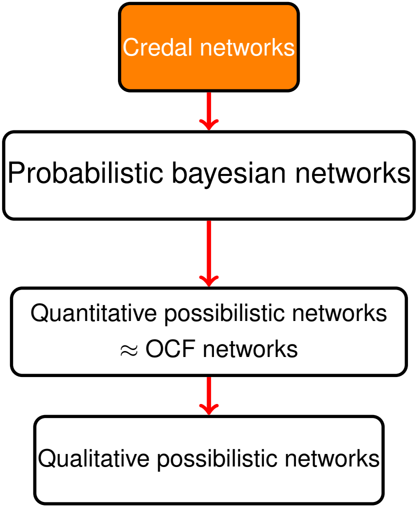

- More expressive languages

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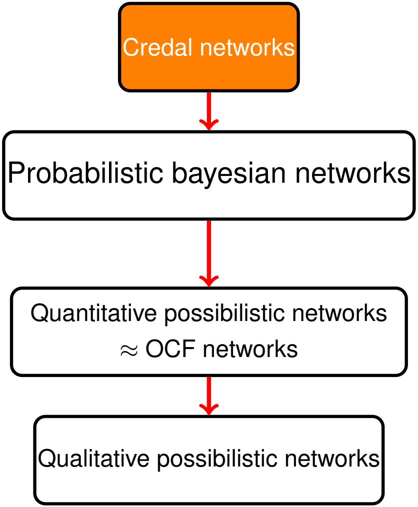

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- An interval-based distribution $I\pi$ is a function from Ω to intervals (or sets).

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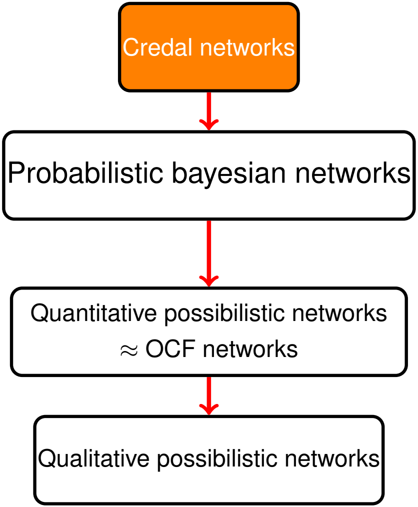

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- $I\pi(\omega) = I$ means that the possibility degree of ω is one of the elements of I.

OCF/Possibilistic networks : Reasoning & Inference

Main queries

Given an uncertainty network and an evidence $E \subseteq \Omega$,

- Plausibility degree of an event : compute $\Pi(Q|E)$.
- Most Plausible Explanation (*MPE*) : compute the most plausible instantiation q of all unobserved variables Q.
- Maximum A Posteriori (*MAP*) : compute the most plausible instantiation q of a subset of the unobserved variables Q.

Probabilistic networks vs OCF/possibilistic networks : Reasoning & Inference

Complexity issues in probabilistic nets [Maua et *al* 2014, de Compos 2011]

	Query	Polytree	Multiply-connected
Bayesian Networks	*Pr*	Polynomial	PP-Complete
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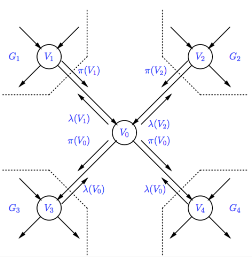
Main inference approaches

Message passing-based

ex. Pearl's belief propagation algorithm.
=> Efficient only on tree-like networks.



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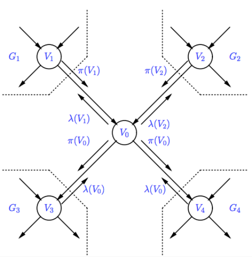

Elimination-based

ex. Variable Elimination, Junction tree algorithm.
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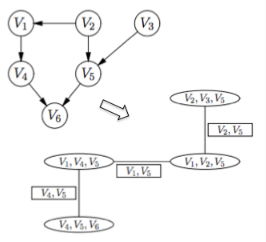

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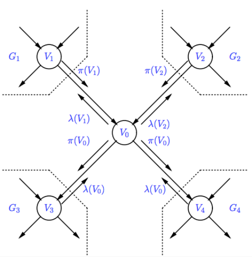

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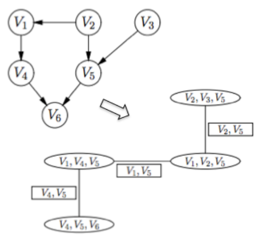

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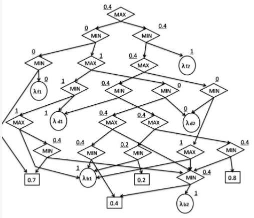
Convert a PN into a target language (ex. Π-DNNF [Ayachi et *al*, 2014]) and answer queries in that language.



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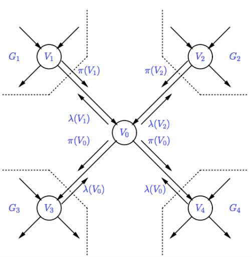

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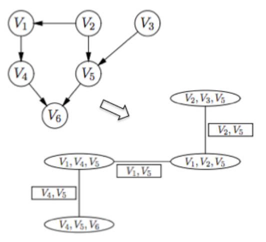

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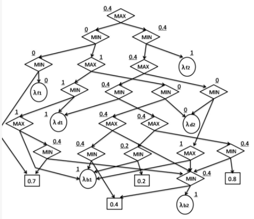

Conditioning
in
OCF/possibility distributions

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OCF/π distribution **+** Input ϕ **=** New OCF/π

- Conditioning = combination + normalisation
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- How to normalize ?

Possibility distribution/OCF : Example

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v	$\neg a$	$\neg h$	α_5

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Input : The conference hotel is fully booked.

Two ways for conditioning

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Minimal change

Visa	A.I.	Hotel	$\pi(\omega)$
v	a	h	0
v	a	$\neg h$	1
$\neg v$	a	$\neg h$	α_2
$\neg v$	$\neg a$	$\neg h$	α_3
v	$\neg a$	h	0
v	$\neg a$	$\neg h$	α_5
$\neg v$	a	h	0
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v	$\neg a$	$\neg h$	α_5
$\neg v$	a	h	α_{10}
$\neg v$	$\neg a$	h	α_{11}

Minimal change

Visa	A.I.	Hotel	$\pi(\omega)$
v	a	h	0
v	a	$\neg h$	1
$\neg v$	a	$\neg h$	α_2
$\neg v$	$\neg a$	$\neg h$	α_3
v	$\neg a$	h	0
v	$\neg a$	$\neg h$	α_5
$\neg v$	a	h	0
$\neg v$	$\neg a$	h	0

Two ways for conditioning

Visa	A.I.	Hotel	$\pi(\omega)$
v	a	h	1
v	a	$\neg h$	α_1
$\neg v$	a	$\neg h$	α_2
$\neg v$	$\neg a$	$\neg h$	α_3
v	$\neg a$	h	α_4
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$\neg v$	a	h	0
$\neg v$	$\neg a$	h	0

Proportional change

Visa	A.I.	Hotel	$\pi(\omega)$
v	a	h	0
v	a	$\neg h$	$\alpha_1/\alpha_1 = 1$
$\neg v$	a	$\neg h$	α_2/α_1
$\neg v$	$\neg a$	$\neg h$	α_3/α_1
v	$\neg a$	h	0
v	$\neg a$	$\neg h$	α_5/α_1
$\neg v$	a	h	0
$\neg v$	$\neg a$	h	0

How to define conditioning interval-based possibility distributions

Three natural requirements for the interval-based conditioning

Degenerate case : If $I\!\pi(\omega)$ contains exactly one single degree $\pi(\omega)$, then the conditioning should coincide with $\pi(.|\phi)$

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$$\forall \phi, I\pi(\omega|\phi) = [\pi(\omega\phi), \pi(\omega|\phi)].$$

Three natural requirements for the interval-based conditioning

Definition

$I\pi$ is said to be more specific than $I\pi'$, denoted $I\pi \subseteq I\pi'$, if

$$I\pi(\omega) \subseteq I\pi'(\omega)$$

holds for all $\omega \in \Omega$

Second postulate

P2. If $I\pi$ is more specific than $I\pi'$ (namely, $I\pi \subseteq I\pi'$)
then $I\pi(.|\phi)$ is more specific than $I\pi'(.|\phi)$ (namely, $I\pi(.|\phi) \subseteq I\pi'(.|\phi)$).

Three natural requirements for the interval-based conditioning

Minimality :

P3. There exist no operation $I\pi(.|_1\phi)$ that satisfies both properties **P1–P2** and for which :

- $I\pi(\omega|_1\phi) \subseteq I\pi(\omega|\phi)$ for all $I\pi$, ω, and ϕ,
- $I\pi(\omega|_1\phi) \neq I\pi(\omega|\phi)$ for some $I\pi$, ω, and ϕ.

Main result

There exists exactly one interval-based conditioning that satisfies the properties **P1–P3**, and which is defined by : $\forall \omega \in \Omega$,

$$I\!\pi(\omega|\phi) = \mathrm{IntCl}(\{\pi(\omega|\phi) : \pi \in \mathcal{C}(I\!\pi)\}) \quad (1)$$

where IntCl is the interval closure.

Interval-based possibility distributions

Compatible distributions

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Family of compatible distributions

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Compatible distributions

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Family of compatible distributions

ω	$I\pi(\omega)$
ω_1	[.7, 1]
ω_2	[.6, .8]
ω_3	[.4, .5]

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ω	$\pi_1(\omega)$
ω_1	1
ω_2	.7
ω_3	.4

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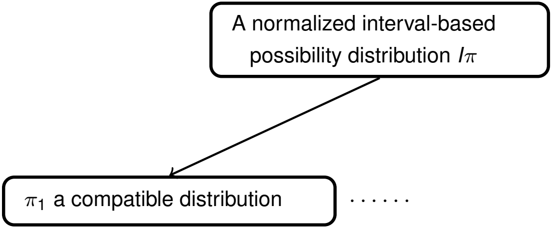
ω_1	1
ω_2	.6
ω_3	.5

ω	$\pi_3(\omega)$
ω_1	1
ω_2	.7
ω_3	**1**

A safe way to define conditioning

A normalized interval-based possibility distribution $I\pi$

A safe way to define conditioning



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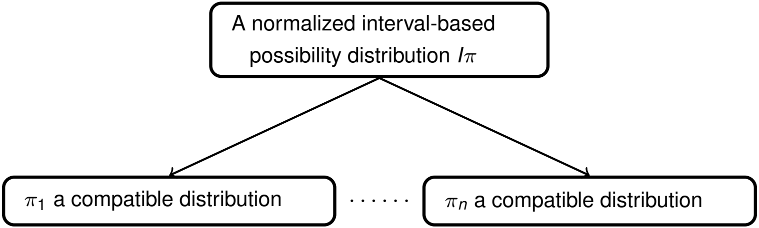

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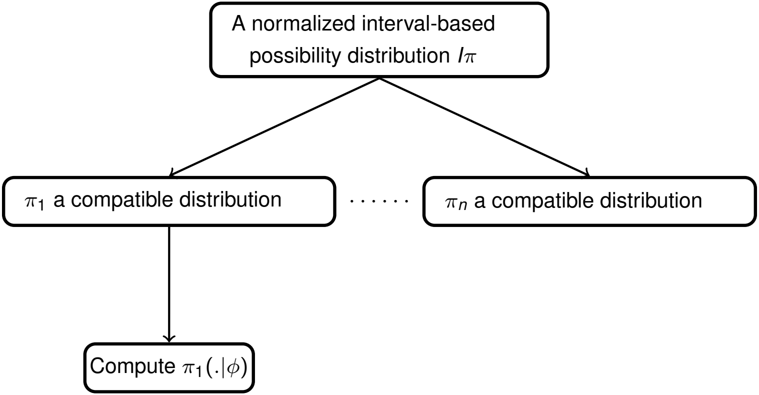

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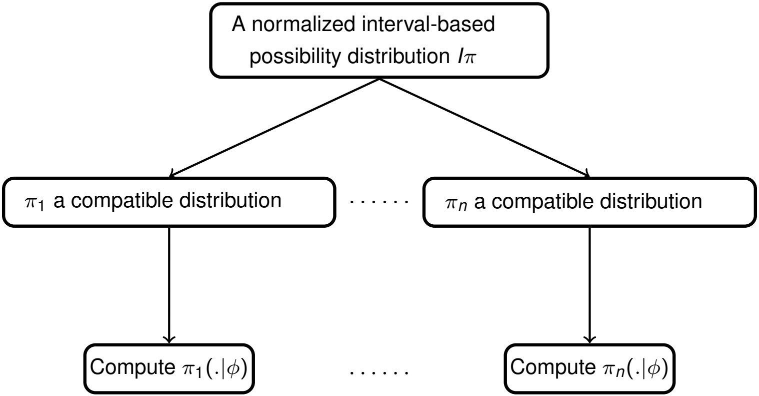

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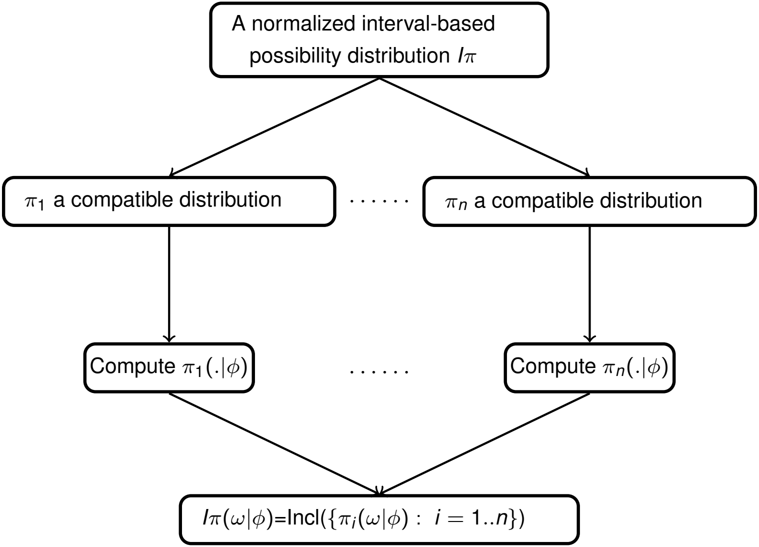


Conditioning under uncertain input
vs
causality

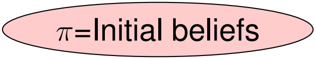
Example : Input (new information)

- Input
  - PC-chair : only papers that deal with AI were accepted
  - The source that supported "booking in a conference hotel" was weakly reliable
  - A normal situation is : "she did not book a conference hotel"
- represented by :

$$\mu = \{(\neg a, 0), (a \wedge h, \alpha_5), (a \wedge \neg h, 1)\}$$

  - $(\neg a, 0)$: all solutions where "a is true" are impossible
  - $(a \wedge h, \alpha_5)$: decrease (if needed) all solutions of $a \wedge h$ to α_5
  - $(a \wedge \neg h, 1)$: sincrease the best solutions until reaching 1

Example : Three parallel and local changes



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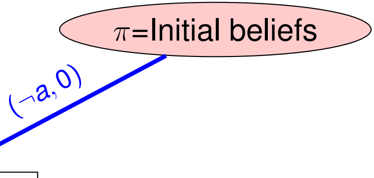

Visa	A.I.	Hotel	$\pi(\omega)$
–	$\neg a$	–	0

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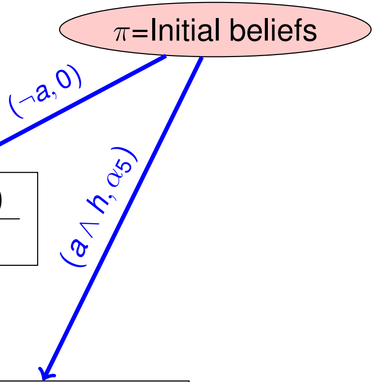

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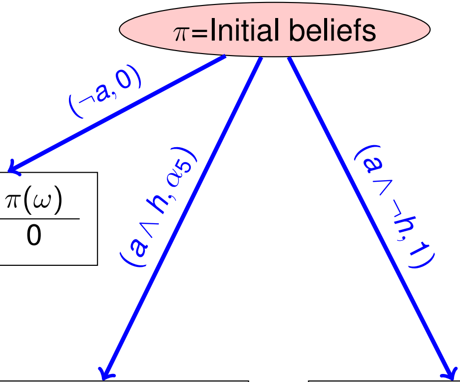

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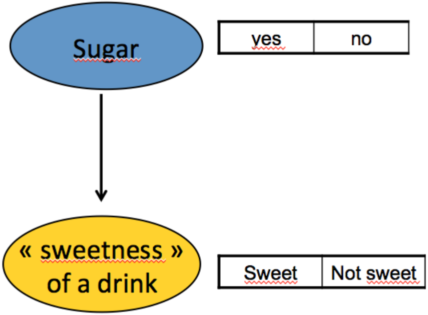
Observations vs Interventions



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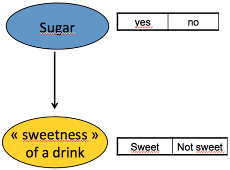

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  belief({Sugar=no})>belief({Sugar=yes}).

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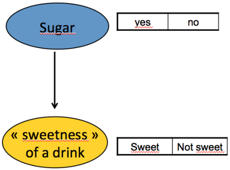

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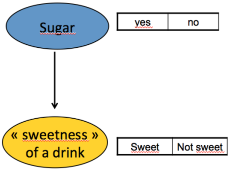

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  $$\text{belief}(\{\text{Sugar=no}\}) > \text{belief}(\{\text{Sugar=yes}\}).$$

- New information : observation (taste of the coffee) :

- This new information changes your beliefs on used ingredient :

  $$\text{belief}(\{\text{Sugar=no}\}) \downarrow$$

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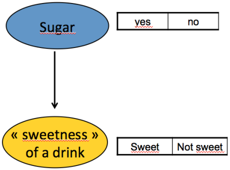

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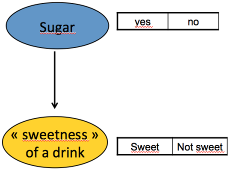

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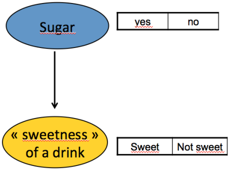

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- Will this new information change your beliefs ?

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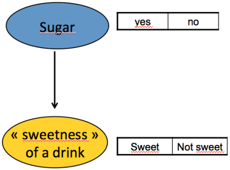

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- Will this new information change your beliefs ?
- Bel (Sugar) will remain unchanged

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- The handling of interventions is more motived by a graphical structure instead of uncertainty frameworks (The Pearl's "do" operator is introduced within OCF framework)
- A causal graph is such that for every A, B in V there is an edge from A to B iff A is a direct cause of B.

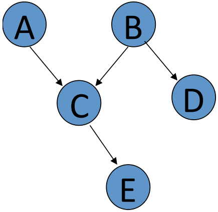
Representing interventions

- The "do" operator for interventions : $\pi'(.) = \pi(.|do(A = a))$
- Natural requirements for $\pi'(.)$:
  - $A = a$ is a sure piece of information : $\Pi'(A \neq a) = 0$.
  - Beliefs on direct causes U_i of "Ai" are unchanged

$$\Pi'(Ai = a, U_i) = \Pi'(U_i) = \Pi'(U_i)$$

  - Minimal change principle :
    - π' is close to π
    - Orders between elements of each partition (Ui) should be preserved
- Intervention = Qualitative counterpart of jeffrey's conditioning with uncertain input

Graph Mutilation



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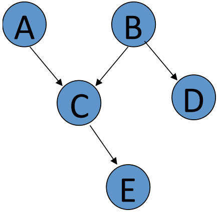


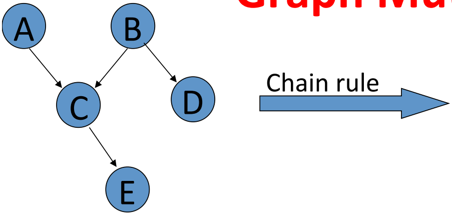
Chain rule →

Joint distribution

π(ABCDE)=

π(E|C)*π(C|AB)*π(D|B)*π(A)*π(B)

Graph Mutilation



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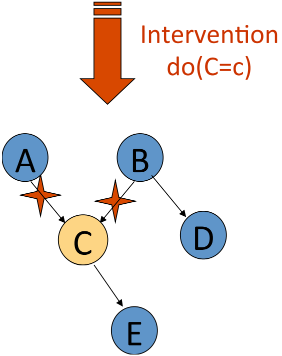

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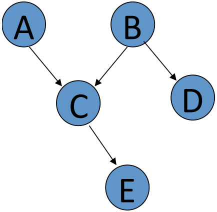

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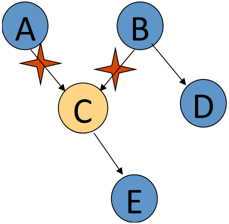
do(C=c)



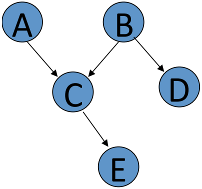
Chain rule

New joint distribution

$\pi'(ABCDE)=0$ if $C\neq c$

$= \pi(E|c)*\pi(D|B)*\pi(A)*\pi(B)$ other.

Graph Mutilation



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Intervention
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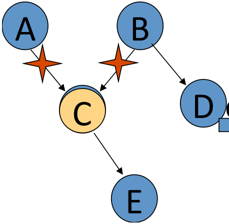
Intervention as revising with uncertain input



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Counterpart of Pearl's mutilated grah

Conclusions

- Brief overview on non-probabilistic networks
- Strong relationships between OCF/possibilistic/credal networks and weighted logics
- Expressivness vs computational complexity.
- Jeffrey's rule : a powerful tool for belief revision
- Non-probabilistic credal networks : MAP needs to be defined
- Causality :
  - Belief changes in uncertainty networks : sequences of observations and interventions
  - Causality ascription